Superconducting junctions perturbed by environmental fluctuation

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A superconducting junctions device is investigated in the case of the environmental perturbation. It is found that a net voltage can be produced, whose absolute value represents a phenomenon of resonance versus the additive noise strength, and may be negative, positive, and zero. By controlling the correlation between the additive and multiplicative noises, a reversal for the net voltage can be induced. The dc voltage versus the dc current is studied. It is shown that (1) with increasing the additive noise strength, the curve of the dc voltage versus the dc current is nearer and nearer to the one for the Ohmic theorem; (2) the behavior of the dc voltage versus the dc current can be manipulated by controlling the noise's strengths. In addition, we study the mean first passage time (MFPT) for the electron pair over a period of the potential, and find that the transition rate (i.e., inverse of the MFPT) can be suppressed by the positive correlations between the additive and multiplicative noises and show a minimum as the function of the noise's strengths.

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I. INTRODUCTION

Recently, there has been an increasing interest in studying the net voltage [1,2] (i.e., a nonzero dc voltage with a zero dc current) and the dc current-voltage characteristcs [3,4] in Josephson junction (superconducting junction) with noises. It is reported that the asymmetric noise can produce net voltage [1,2], and dc voltage rectification [3,4]. We have showed that the correlated symmetric noise can also produce a net voltage [2,5], which stems from a symmetry breaking of the system induced by the correlation between the additive and multiplicative noises.

However, all of the above work was focused on the single Josephson junction. In Ref. [6], Zapata *et al.*, investigated the dc current-voltage characteristics for an asymmetric dc device with three Josephson junctions (depicted in Fig. 1) threaded by a magnetic flux and driven by periodic signal and additive noises. But they did not consider the case in the presence of the additive and multiplicative noises, especially the case when the additive and multiplicative noises are correlated. In this paper, we will study the net voltage, the dc current-voltage characteristics, and the mean first passage time for this device in the case of the environmental perturbation, together with the thermal fluctuation. (The environmental perturbation can be described by the multiplicative noises in the Langevin equation [7–9], and the thermal fluctuation by the additive Gaussian white noise [10].)

We focus on this device (to see Fig. 1) formed by Josephson junctions whose phase is a classical variable and which can be adequately described by the "resistively shunted junction" model [11,12]. Thus, the phase ϕ_i across Josephson junction i on the left arm obeys the equation ($i=1,2$)

$$I_l(t)=J_i\sin(\phi_i)+\frac{\hbar}{2eR_i}\dot{\phi}_i+\frac{\hbar C_i}{2e}\ddot{\phi}_i, \tag{1}$$

where $I_l(t)$ is the current through the left arm, and R_i, C_i, and J_i are the resistance, capacitance, and critical current of junction i. For simplicity, we assume here that the two junctions in series are identical. We take $C_1=C_2=2C_l$, $R_1=R_2=R_l/2$, and $J_1=J_2=J_l$. The total voltage drop across the two junctions is $V=V_1+V_2$, where $V_i=(\hbar/2e)\dot{\phi}_i$. If $\phi_1(t)$ is a solution for the first junction, then $\phi_2(t)=\phi_1(t)=\phi_l(t)/2$ is also a solution for the second junction [13]. This implies $V=\dot{\phi}_l\hbar/2e$ with ϕ_l satisfying the equation,

$$I_l(t)=J_l\sin(\phi_l/2)+\frac{\hbar}{2eR_l}\dot{\phi}_l+\frac{\hbar C_l}{2e}\ddot{\phi}_l. \tag{2}$$

Hence, a series of two identical Josephson junctions can be described by the same equation as a single junction, with the

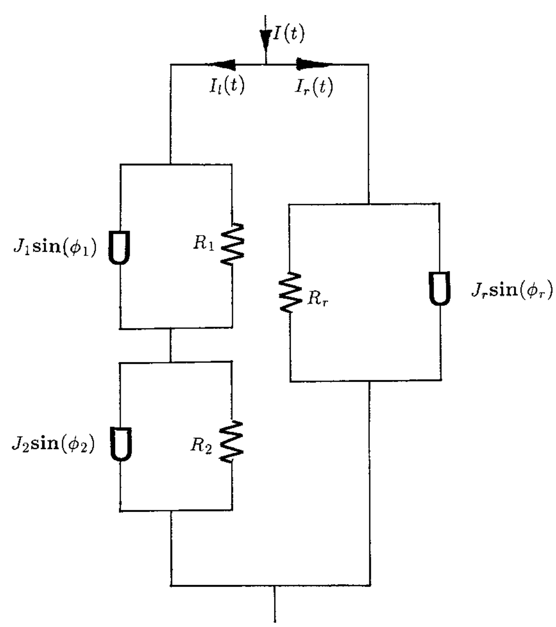

FIG. 1. The three-Josephson-junction device.

only difference that in the sine function: the argument $\phi/2$ ($\phi=\phi_l$) occurs [14]. On the right arm, the phase across the single junction obeys an equation that reads as in Eq. (1) with the labels l and i replaced by r. In the following, we assume that each Josephson link operates in the overdamped limit, $(2e/\hbar)J_\alpha R_\alpha^2 C_\alpha \ll 1$ ($\alpha=l,r$), so that the capacitive terms can be neglected in Eq. (2) as well as in its right arm counterpart [12,15].

The total current through the device is $I(t)=I_l(t)+I_r(t)$, which is marked in Fig. 1. In the limit, where the total loop inductance $L=L_l+L_r$ are $|LI(t)|\ll\Phi_0=h/(2e)$, the total magnetic flux Φ is approximately the external flux Φ_e. Then, the phase around the loop yields $\phi_l-\phi_r=-\phi_e+2\pi n$ with $\phi_e=2\pi\Phi_e/\Phi_0$. So the phase satisfies the equation

$$\frac{\hbar}{eR}\dot{\phi}=-J_l\sin\frac{\phi}{2}-J_r\sin(\phi+\phi_e)+I(t), \tag{3}$$

in which $\phi=\phi_l$.

II. THE MODEL AND ITS FOKKER-PLANCK EQUATION

In the case of the environmental perturbation, such as the external vibration, the change of the external temperature, the perturbation of the external electromagnetic fields, and so on, the internal structure of the Josephson junction should change. In general, the change is very small. But when the environmental perturbation becomes larger and larger, the change will become clearer and clearer. The change of the internal structure of the Josephson junction will vary the critical electric current. Now we describe these fluctuations by stochastic external parameters $J_l+\sigma_1\xi_1^{(0)}(t)$ and $J_r+\sigma_2\xi_2^{(0)}(t)$, in which $\xi_1^{(0)}(t)$ and $\xi_2^{(0)}(t)$ are assumed to be the stochastic forces of the Gaussian white noise, σ_1 and σ_2 are positive constants. On the other hand, the stochastic driving current is taken to be the thermal Gaussian white noise $I(t)=\eta_0(t)$. Then, Eq. (3) becomes [2,4,5]

$$\frac{\hbar}{eR}\dot{\phi}=-[J_l+\sigma_1\xi_1^{(0)}(t)]\sin\frac{\phi}{2}-[J_r+\sigma_2\xi_2^{(0)}(t)]\sin(\phi+\phi_e)+\eta_0(t), \tag{4}$$

where the noises $\xi_1^{(0)}(t)$, $\xi_2^{(0)}(t)$, and $\eta_0(t)$ have zero mean, and autocorrelation functions $\langle\xi_1^{(0)}(t)\xi_1^{(0)}(t')\rangle_f=2D_1\delta(t-t')$, $\langle\xi_2^{(0)}(t)\xi_2^{(0)}(t')\rangle_f=2D_2\delta(t-t')$, and $\langle\eta_{(0)}(t)\eta_{(0)}(t')\rangle_f=2D\delta(t-t')$ ($\langle\rangle_f$ denotes the average over noise). The multiplicative noises $\xi_1^{(0)}(t)$ and $\xi_2^{(0)}(t)$, and the additive noise $\eta_0(t)$ in this three-superconducting-junction device come from the external environmental perturbation and the thermal fluctuation, respectively. However, they are not independent but related to each other, since the environmental perturbation can also lead to a change of the thermal vibration of molecules in the Josephson junction. Thus, we should consider the correlations between the additive and multiplicative noises and assume that their correlation functions are taken to be the simple relations [2,4,5], $\langle\eta_0(t)\xi_1^{(0)}(t')\rangle_f=2\lambda_1\sqrt{DD_1}\delta(t-t')$, $\langle\eta_0(t)\xi_2^{(0)}(t')\rangle_f=2\lambda_2\sqrt{DD_2}\delta(t-t')$, and $\langle\xi_1^{(0)}(t)\xi_2^{(0)}(t')\rangle_f=0$, with $-1\le\lambda_1\le1$ and $-1\le\lambda_2\le1$ indicating the strengths of the correlations. Equation (4) can be rewritten as

$$\dot{\phi}=f(\phi)-\xi_1(t)\sin(\phi/2)-\xi_2(t)\sin(\phi+\phi_e)+\eta(t), \tag{5}$$

where $f(\phi)=-\omega_l\sin(\phi/2)-\omega_r\sin(\phi+\phi_e)$, $\omega_l=eRJ_l/\hbar$, $\omega_r=eRJ_r/\hbar$, $\xi_1(t)=(e\sigma_1R/\hbar)\xi_1^{(0)}(t)$, $\xi_2(t)=(e\sigma_2R/\hbar)\xi_2^{(0)}(t)$, and $\eta(t)=(eR/\hbar)\eta_0(t)$. The Stratonovich interpretation of the stochastic differential equation (5) yields the Fokker-Planck equation [10,16–18]

$$\partial_tP(\phi,t)=-\partial_\phi A(\phi)P(\phi,t)+\partial_\phi^2B(\phi)P(\phi,t), \tag{6}$$

where

$$\begin{aligned}A(\phi)=&-\omega_l\sin(\phi/2)-\omega_r\sin(\phi+\phi_e)+(\tilde{D}_1/4)\sin\phi\\&+(\tilde{D}_2/2)\sin[2(\phi+\phi_e)]-(\lambda_1\sqrt{\tilde{D}\tilde{D}_1}/2)\cos(\phi/2)\\&-(\lambda_2\sqrt{\tilde{D}\tilde{D}_2})\cos(\phi+\phi_e),\end{aligned}$$

and

$$\begin{aligned}B(\phi)=&\tilde{D}_1\sin^2(\phi/2)+\tilde{D}_2\sin^2(\phi+\phi_e)+\tilde{D}\\&-2\lambda_1\sqrt{\tilde{D}\tilde{D}_1}\sin(\phi/2)-2\lambda_2\sqrt{\tilde{D}\tilde{D}_2}\sin(\phi+\phi_e)\end{aligned}$$

with $\tilde{D}_1=(e\sigma_1R/\hbar)^2D_1$, $\tilde{D}_2=(e\sigma_2R/\hbar)^2D_2$, and $\tilde{D}=(eR/\hbar)^2D$.

III. THE NET VOLTAGE

The net voltage is given by (for convenience, we make the calculation in a dimensionless form, and set $\hbar/e=\sigma_1=\sigma_2=R=1$)

$$\begin{aligned}\langle V(t)\rangle&=\langle\langle V(\phi,t)\rangle_\phi\rangle_f\\&=\langle\langle\dot{\phi}\rangle_\phi\rangle_f\\&=\langle\langle-\omega_r\sin(\phi/2)-\omega_l\sin\phi-\xi_1(t)\sin(\phi/2)\\&\quad-\xi_2(t)\sin\phi\rangle_f\rangle_\phi,\end{aligned} \tag{7}$$

where $\langle\rangle_\phi$ stands for the average over ϕ. According to the Novikov theorem [19], we have

$$\langle\xi_1(t)\sin(\phi/2)\rangle_f=(D_1/2)\cos(\phi/2)\times[-\sin(\phi/2)+\lambda_1\sqrt{D/D_1}],$$

$$\langle\xi_2(t)\sin(\phi+\phi_e)\rangle_f=D_2\cos(\phi+\phi_e)\times[-\sin(\phi+\phi_e)+\lambda_2\sqrt{D/D_2}].$$

Under periodic boundary conditions, the stationary solution of Eq. (6) is [10,18]

$$P_s(\phi)=N\frac{e^{\Phi(\phi)}}{B(\phi)}\int_0^{4\pi}d\phi' e^{-\Phi(\phi')-\Phi(4\pi)\theta(\phi-\phi')}. \quad (8)$$

Here, $\Phi(\phi)=\int_0^{\phi}[A(\phi')/B(\phi')]d\phi'$, $\theta(\phi-\phi')$ is the Heaviside step function and N is a normalized constant.

From Eqs. (7) and (8), we obtain

$$\begin{aligned}\langle V\rangle_s&=\lim_{t\to\infty}\frac{1}{t}\int_0^t\langle\langle V(\phi,\tau)\rangle_\phi\rangle_f d\tau\\&=\langle A(\phi)\rangle_\phi\\&=N\int_0^{2\pi}d\phi\frac{A(\phi)e^{\Phi(\phi)}}{B(\phi)}\int_0^{2\pi}d\phi' e^{-\Phi(\phi')-\Phi(2\pi)\theta(\phi-\phi')}.\end{aligned} \quad (9)$$

In Fig. 2, we plot the net voltage versus the additive noise strength D from Eq. (9) in the dimensionless form [Fig. 2(a) corresponds to the net voltage versus the additive noise strength for different values of the critical current J_l ($J_l=0.3$, 0.5, 0.7, and 1, respectively) with $J_r=1$, $D_1=D_2=0.3$, and $\lambda_1=\lambda_2=0.3$; Fig. 2(b) to that for different values of J_r (J_r=0.2, 0.5, 0.7, and 1, respectively) with $J_l=1$, $D_1=D_2=0.3$, and $\lambda_1=\lambda_2=0.3$; Fig. 2(c) to that for different values of λ_1 ($\lambda_1=-0.9$, -0.5, 0.5, and 0.9, respectively) with $J_l=J_r=1$, $D_1=D_2=0.3$, and $\lambda_2=0.3$; and Fig. 2(d) to that for different values of λ_2 ($\lambda_1=-0.9$, -0.5, 0.5, and 0.9, respectively) with $J_l=J_r=1$, $D_1=D_2=0.3$, and $\lambda_1=0.3$]. From the figures, we can find that the net voltage is a nonmonotonic function of the additive noise strength and its absolute value has a clear peak (a manifestation of the phenomenon of resonance for the absolute value of the net voltage), and it may be negative, positive, or zero. In Refs. [2,5], we studied the single superconducting junction in the case of environmental perturbation. It is found that the net voltage is negative. In this paper, we find that for the Josephson junctions device the net voltage can be negative, positive, or zero. From Figs. 2(c) and 2(d), we note that a reversal for the net voltage can be induced by controlling the correlations between the additive and multiplicative noises.

Here, we wish to give some explanation for the origin of the net voltage. First, in the absence of the multiplicative noises, no net voltage can be produced. A nonzero net voltage with $\xi_i(t)=0$ means that only thermal fluctuation is converted into work and implies a violation of the second law of thermodynamics. Second, in the presence of multiplicative noises, (1) if the function $f(\phi)$ is symmetric and $\lambda_1,\lambda_2\neq 0$, the net voltage can be caused, due to the symmetry breaking induced by the correlations between the additive and multiplicative noises [5]; (2) if $f(\phi)$ is asymmetric, even if $\lambda_1=\lambda_2=0$, there is still the nonzero net voltage [5]. Thus, the correlations between the additive and multiplicative noises, or the asymmetry of $f(\phi)$ [or the asymmetry of the potential, which is $U(\phi)=-\int^{\phi}f(\phi')d\phi'$] is ingredient for producing the net voltage for model (4). The reason for pro-

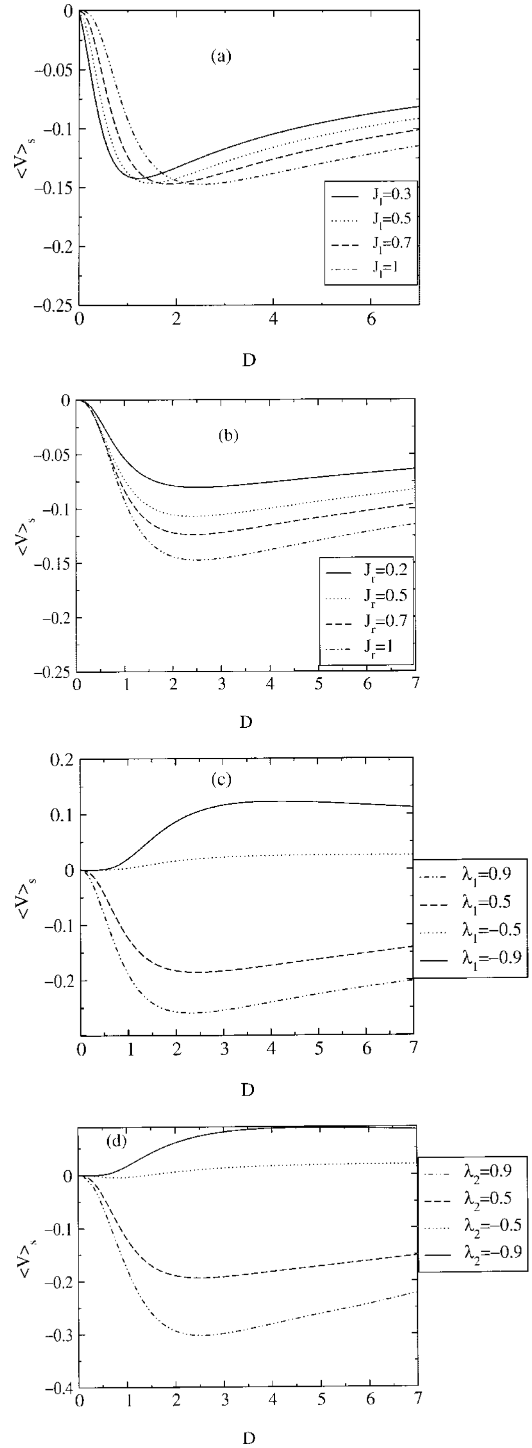

FIG. 2. The net voltage versus the additive noise strength in the dimensionless form. Figure 2(a) corresponds to the net voltage versus the additive noise strength for different values of the critical current J_l ($J_l=0.3$, 0.5, 0.7, and 1, respectively) with $J_r=1$, $\phi_e=\pi/2$, $D_1=D_2=0.3$, and $\lambda_1=\lambda_2=0.3$; Fig. 2(b) to that for different values J_r ($J_r=0.2$, 0.5, 0.7, and 1, respectively) with $J_l=1,\phi_e=\pi/2$, $D_1=D_2=0.3$, and $\lambda_1=\lambda_2=0.3$; the Fig. 2(c) to that for different values λ_1 ($\lambda_1=-0.9$, -0.5, 0.5, and 0.9, respectively) with $J_l=J_r=1,\phi_e=\pi/2$, and $D_1=D_2=0.3$, and $\lambda_2=0.3$; Fig. 2(d) to that for different values λ_2 ($\lambda_2=-0.9$, -0.5, 0.5, and 0.9, respectively) with $J_l=J_r=1,\phi_e=\pi/2$, $D_1=D_2=0.3$, and $\lambda_1=0.3$.

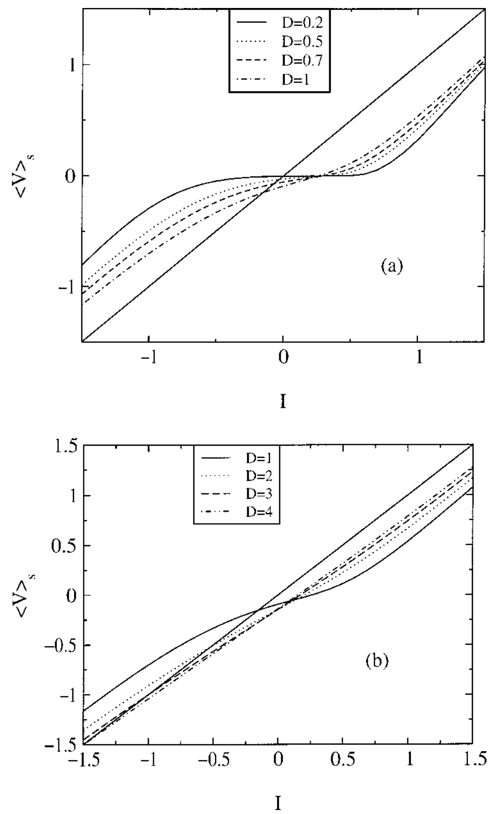

FIG. 3. The dc voltage versus the dc current in the dimensionless form. Fig. 3(a) corresponds to $D=0.2$, 0.5, 0.7, and 1, and Fig. 3(b) to $D=1$, 2, 3, and 4 (the diagonal line is plotted by the Ohmic theorem) with $D_1=D_2=0.3$, $\phi_e=\pi/2$, $\lambda_1=\lambda_2=0.3$, and $J_l=J_r=1$.

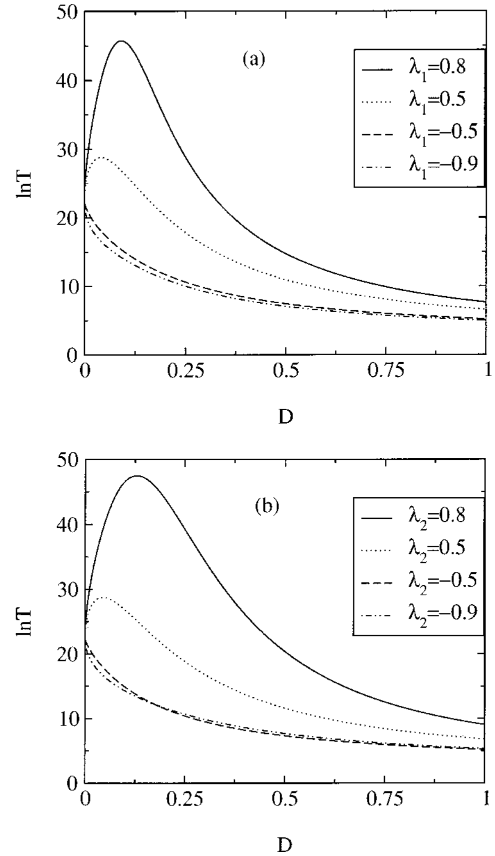

FIG. 4. The natural logarithm (ln) of the MFPT versus the additive noise strength in the dimensionless form for different values of λ_1 [Fig. 4(a), $\lambda_1=-0.9$, -0.5, 0.5, and 0.8 with $\lambda_2=0.3$] and λ_2 [Fig. 4(b), $\lambda_1=-0.9$, -0.5, 0.5, and 0.8 with $\lambda_1=0.3$] with $D_1=D_2=0.3$, $\phi_e=\pi/2$, and $J_l=J_r=1$.

ducing the net voltage is that the symmetry of the system is broken by the function $f(\phi)$, and the correlations between the additive and multiplicative noises. Now the asymmetry of the system makes the probability of the fluctuations on the two sides of the potential barrier different, so that a net voltage arises. The energy in response to the net voltage stems from the noise's energy.

In addition, the phenomenon of resonance for the absolute value of the net voltage happening here should be analyzed. In the Figs. 2(a)–2(d), the additive noise plays twofold roles. On one hand, it stimulates directional motion in response to the asymmetric condition of the system. On the other hand, it reduces the asymmetry of the system which is the root of directional motion. The competition of these two seemly opposite roles makes the curve produce a peak in which a phenomenon of resonance appears for the absolute value of the net voltage.

The dc current-voltage characteristics can be calculated by Eq. (9) if $A(\phi)+I$ [now $I(t)=\eta(t)+I$] is used instead of $A(\phi)$ in Eq. (9) (to include the $A(\phi')$ in $\Phi(\phi)=\int_0^{\phi}[A(\phi')/B(\phi')]d\phi'$). In Figs. 3(a) and 3(b), we plot the dc current-voltage characteristics for different values of the additive noise strength in the dimensionless form [Fig. 3(a) corresponds to $D=1$, 0.7, 0.5, and 0.2, and Fig. 3(b) to $D=4$, 3, 2, and 1 (in the figures, the dashed line is plotted with the Ohmic theorem)]. From these figures, we note that with the increase of the additive noise's strength, the curve for the dc voltage versus the dc current is nearer and nearer to the one for the Ohmic theorem. This is the result of large additive noise or any large perturbation, because in this case the particle does not "feel" the form of the nonlinear potential and the dependence becomes linear. Now one can manipulate the behavior of the dc voltage versus the dc current by controlling the additive noise's strength. For example, one can appropriately adjust the temperature to fit the features of the dc voltage and the dc current to one's demands by taking into account that the thermal additive noise strength D is proportional to the temperature.

In addition, further study shows that when the dc current is not zero, the absolute value of the dc voltage versus the additive noise strength presents the same phenomenon of resonance as in Figs. 2(a)–2(d).

IV. THE MEAN FIRST PASSAGE TIME

In this section, we will calculate the mean first passage time (MFPT) for a particle (an electron pair) over a period of the potential. We start with the particle at $\phi=0$, so the initial condition is $P(\phi,0)=\delta(\phi)$. The boundary condition for the reflecting ($\phi=0$) and absorbing ($\phi=4\pi$) boundaries, respectively, are $\partial_\phi P(\phi,t)|_{\phi=0}=0$ and $P(\phi,t)|_{\phi=4\pi}=0$. Below we will calculate the MFPT over a period in the dimensionless form (for convenience, we set $\hbar/e=\sigma_1=\sigma_2=R=1$).

The MFPT is defined as [10]

$$T(\phi)=-\int_0^\infty t\partial_t G(\phi,t)dt=\int_0^\infty G(\phi,t)dt, \qquad (10)$$

in which $G(\phi,t)$ is the probability density for the backward Fokker-Planck equation of Eq. (5). From Eq. (6), we can get the backward Fokker-Planck equation for Eq. (5) as

$$\partial_t G(\phi,t)=A(\phi)\partial_\phi G(\phi,t)+B(\phi)\partial_\phi^2 G(\phi,t), \qquad (11)$$

where

$$\begin{aligned}A(\phi)=&-\omega_l\sin(\phi/2)-\omega_r\sin(\phi+\phi_e)+(D_1/4)\sin\phi\\&+(D_2/2)\sin[2(\phi+\phi_e)]-(\lambda_1\sqrt{DD_1}/2)\cos(\phi/2)\\&-(\lambda_2\sqrt{DD_2})\cos(\phi+\phi_e)\end{aligned}$$

and

$$\begin{aligned}B(\phi)=&D_1\sin^2(\phi/2)+D2\sin^2(\phi+\phi_e)+D\\&-2\lambda_1\sqrt{DD_1}\sin(\phi/2)-2\lambda_2\sqrt{DD_2}\sin(\phi+\phi_e).\end{aligned}$$

From Eqs. (10) and (11), we obtain the equation of the MFPT

$$A(\phi)\partial_\phi T(\phi)+B(\phi)\partial_\phi^2 T(\phi)=-1. \qquad (12)$$

Here, the reflecting boundary condition is $\partial_\phi T(0)=0$ and the absorbing boundary condition $T(4\pi)=0$. The MFPT for the particle over a period that starts at $\phi=0$ is $T=T(0)$. The escape rate for a particle over a period is $\gamma=1/T$.

According to Eq. (12), we investigate the activation of MFPT as the function of the noise strength for different values of the correlations between the additive and multiplicative noises. We find that the escape rate (i.e., the reciprocal value of the MFPT) over a period of the potential can be suppressed by the positive correlations and show a minimum as the function of the three noise's strengths. This phenomenon has been reported in Refs. [20,21]. It is called the ''giant suppression of the activation rate'' (GS). In Figs. 4(a) and 4(b), we plot the natural logarithm of the MFPT over a period of the potential versus the additive noise strength in the dimensionless form for different values of λ_1 and λ_2, respectively. The figures show that the MFPT curves for positive correlation exhibits a peak value (i.e., GS exits), while curves for negative correlations do not, and the larger the correlation is, the higher the peak becomes. Now the positive correlation becomes more suppressive on the activation as the correlation grows, which is exactly the main conclusion of Ref. [20]. This is because when the correlations are positive the instantaneous barrier can be lifted up (the negative correlations case is just the contrary).

ACKNOWLEDGMENT

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